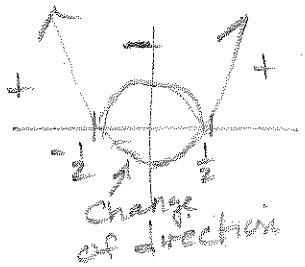
② $4x^2-1 \geq 0$

$(2x+1)(2x-1) \geq 0$

$x = -\frac{1}{2}, \frac{1}{2}$

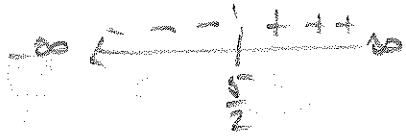
$$f(x) = \begin{cases} 4x^2-1, & x \leq -\frac{1}{2} \text{ or } x \geq \frac{1}{2} \\ -(4x^2-1), & -\frac{1}{2} < x < \frac{1}{2} \end{cases}$$

$$f(x) = \begin{cases} 4x^2-1, & \text{if } -\frac{1}{2} \geq x \geq \frac{1}{2} \\ -(4x^2-1), & \text{if } -\frac{1}{2} < x < \frac{1}{2} \end{cases}$$



① $2x-5 \geq 0$

$x \geq \frac{5}{2}$

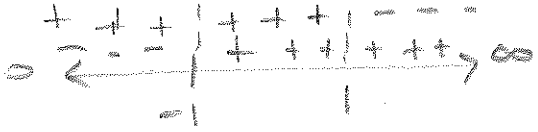


$$f(x) = \begin{cases} 2x-5, & x \geq \frac{5}{2} \\ -(2x-5), & x < \frac{5}{2} \end{cases}$$

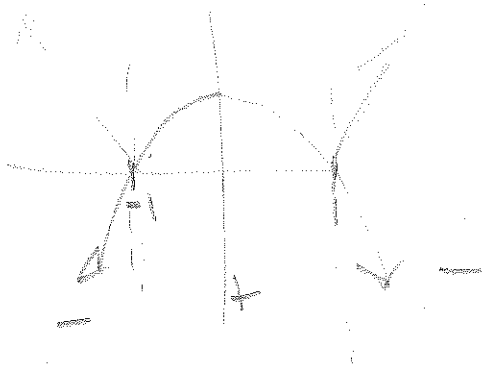
③ $1-x^2 \geq 0$

$(1+x)(1-x) \geq 0$

$x = -1, 1$



$$f(x) = \begin{cases} 1-x^2, & -1 \leq x \leq 1 \\ -(1-x^2), & -1 > x > 1 \end{cases}$$



④ $f\left(\frac{1}{x+4}\right) = \left(\frac{1}{x+4}\right)^2 + 3\left(\frac{1}{x+4}\right) + 1$

$$= \frac{1}{(x+4)^2} + \frac{3(x+4)}{(x+4)^2} + \frac{(x+4)^2}{(x+4)^2}$$

$$= \frac{1 + 3x + 12 + x^2 + 8x + 16}{(x+4)^2}$$

$$= \frac{x^2 + 11x + 29}{(x+4)^2}$$

⑤ $f(x) = x^2 + 3x + 1$

$f(x+h) = (x+h)^2 + 3(x+h) + 1$

$= x^2 + 2xh + h^2 + 3x + 3h + 1$

⑥ $g(x) = \frac{1}{x+4}$

$x = \frac{1}{y+4}$

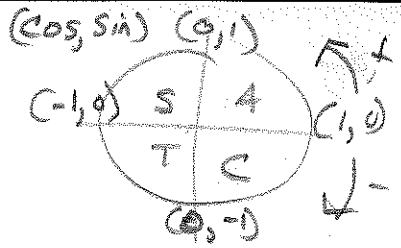
$y+4 = \frac{1}{x}$

$y = \frac{1}{x} - 4 = \frac{1-4x}{x}$

$g^{-1}(x) = \frac{1-4x}{x}$

(7) $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

(8) $\cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$



(9) $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

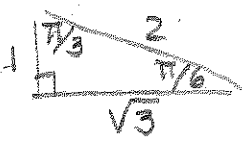
(10) $\sin 5\pi = 0$

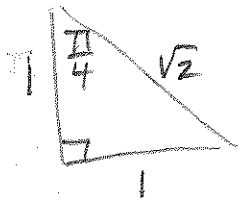
(13) $\sec \frac{2\pi}{3} = \frac{1}{\cos \frac{2\pi}{3}}$

$= -\frac{1}{\frac{1}{2}} = -2$

(14) $\cot \frac{11\pi}{6} = \frac{1}{\tan \frac{11}{6}\pi}$

$= -\frac{1}{\frac{1}{\sqrt{3}}} = -\sqrt{3}$





(11) $\cos\left(\frac{-2\pi}{3}\right) = -\frac{1}{2}$

(12) $\csc \frac{3\pi}{2} = \frac{1}{\sin \frac{3\pi}{2}}$

$= \frac{1}{-1} = -1$

(15) $\csc \pi = \frac{1}{\sin \pi} = \frac{1}{0}$

= undefined

(16) $f(-2) = 2(-2)+1$
$= -3$

(17) $f(25) = \sqrt{25}$
$= 5$

(18) $f(-9) = 2(-9)+1$
$= -17$

(19) Solve for y

$3xy + 2y = -6x + 1$

$y(3x+2) = -6x+1$

$y = \frac{-6x+1}{3x+2}$

(20) $3y^2 + 14x = 2x^2$

$3y^2 = 2x^2 - 14x$

$y^2 = \frac{2x^2-14x}{3}$

$y = \pm\sqrt{\frac{2x^2-14x}{3}}$

(21) $9x - 3y + 12xy = 7x^2 + 4y$

$2xy - 7y = 7x^2 - 9x$

$y(2x-7) = 7x^2 - 9x$

$y = \frac{x(7x-9)}{2x-7}$

(22) $12x^2y + 70xy - 3x^3 + 12x = 9y + 13x^2$

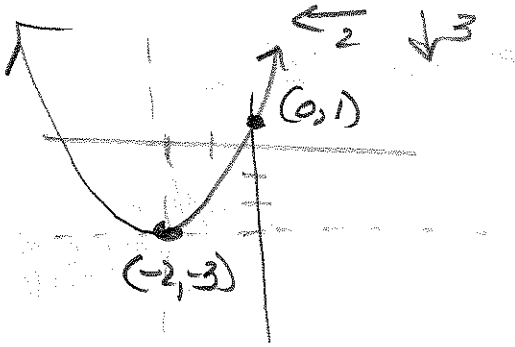
$12x^2y + 70xy - 9y = 13x^2 + 3x^3 - 12x$

$y(12x^2 + 70x - 9) = 3x^3 + 13x^2 - 12x$

$$y = \frac{x(3x^2 + 13x - 12)}{12x^2 + 70x - 9}$$

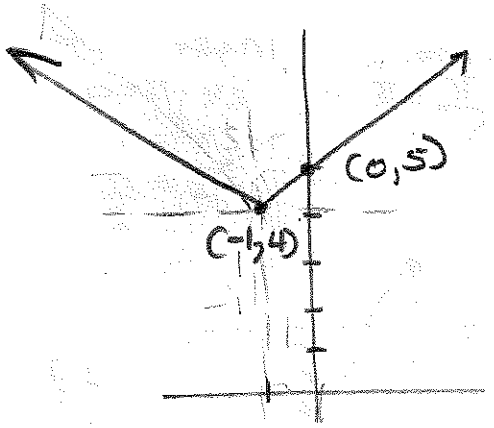
(23) $f(x) = (x+2)^2 - 3$

$\leftarrow 2 \quad \downarrow 3$



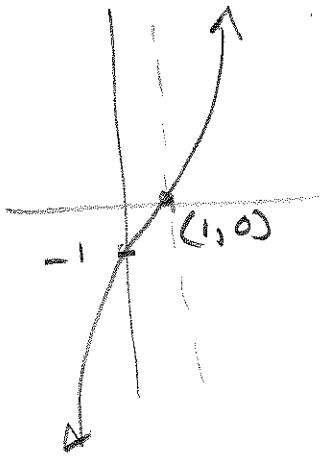
(24) $F(x) = |x+1| + 4$

$\leftarrow 1 \quad \uparrow 4$



(25) $f(x) = (x-1)^3$

$\rightarrow 1$



(26) $f(x) = -\sqrt{x-5} + 2$ (down)

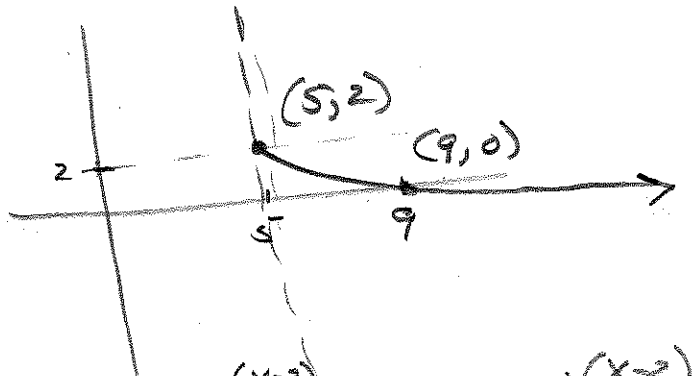
$\rightarrow 5 \quad \uparrow 2$

$2 = \sqrt{x-5}$

$4 = x - 5$

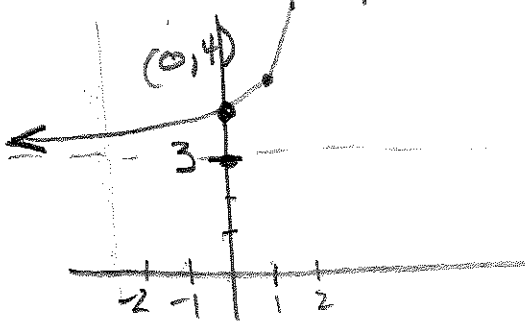
$x = 9$

$(9, 0)$



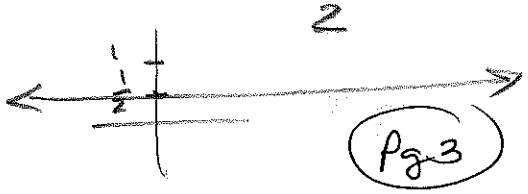
(27) $f(x) = 2^x + 3$

$\uparrow 3$

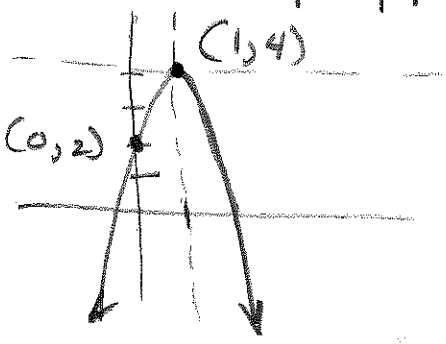


(28) $F(x) = \frac{1}{2}^{(x-2)} \neq \left(\frac{1}{2}\right)^{(x-2)}$

$$f(x) = \frac{1^{(x-2)}}{2}$$ ← numerator is always 1 ⇒ Constant function

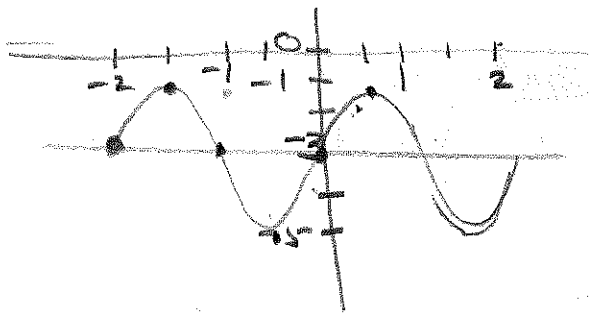


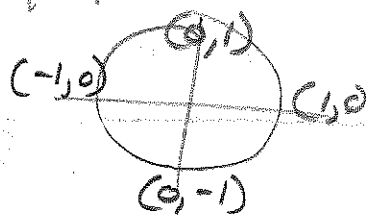
(29) $f(x) = -2(x-1)^2 + 4$ (down; → 1, ↑ 4)



(30) $f(x) = 2\sin \pi x - 3$ (↓ 3)

x	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2
f(x)	-3	-1	-3	-5	-3	-1	-3	-5	-3

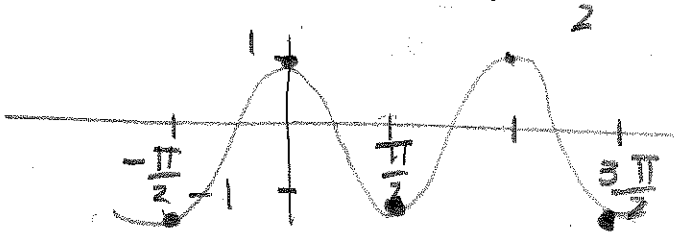




(31) $f(x) = -\cos(2x - \pi)$ (Bottom)

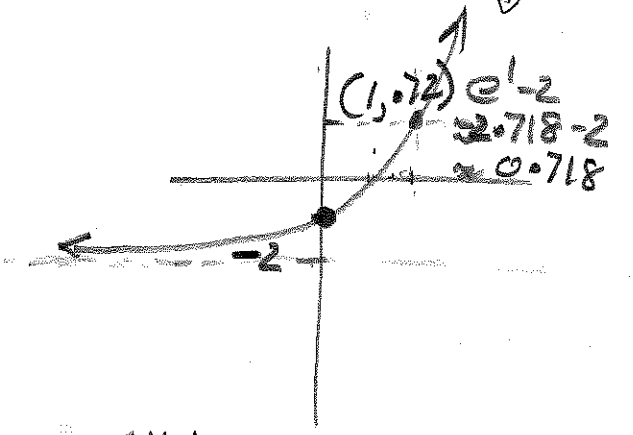
period $\frac{2\pi}{2} = \pi$

phase shift
$2x - \pi = 0$
$2x = \pi$
$x = \frac{\pi}{2}$

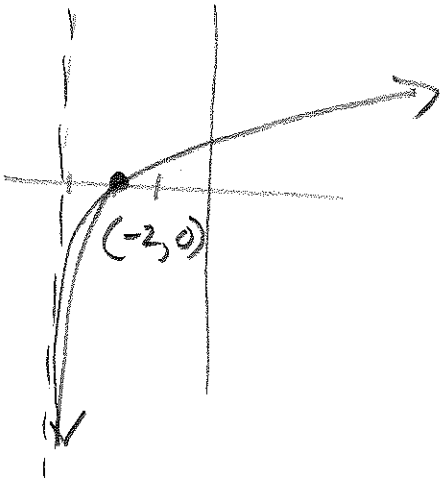


(32) $f(x) = e^x - 2$ (↓ 2)

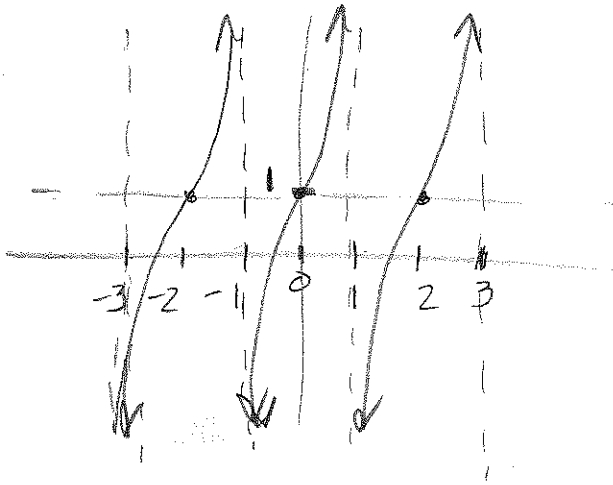
$(1, .72)$ $e^1 - 2$
$2.718 - 2$
≈ 0.718

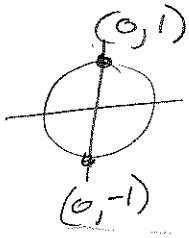


(33) $f(x) = \ln(x+3)$ (← 3)



(34) $f(x) = \tan\left(\frac{x}{2}\right) + 1$ (↑ 1)





tan is undefined at $\frac{\pi}{2} + n\pi$

(35) $f(x) = \dfrac{x}{x^3-3}$

$\dfrac{x}{x^3}$ ← higher degree

$y = 0$ H.A.

$x^3 - 3 = 0$

$x = \sqrt[3]{3}$ V.A.

$\dfrac{x}{x^3-3} = 0$

$x = 0$ $(0,0)$

x-intercept

(36) $f(x) = \dfrac{x^3+2x^2-x-2}{x^2+x-6}$

No H.A.

numerator has higher degree than denominator

$x^2+x-6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

V.A.

$$x^2+x-6 \;\overline{)\; x^3+2x^2-x-2}$$

$x+1$

$\ominus\, x^3 \mp x^2 \pm 6x$

x^2+5x-2

$\ominus\, x^2 \mp x \pm 6$

$4x+4$

Slant asymp

$y = x+1$

$x^2(x+2) - 1(x+2)$

$\dfrac{(x^2-1)(x+2)}{(x+3)(x-2)} = 0$

$(x^2-1)(x+2) = 0$

$x = \pm 1, -2$

x-intercepts

$(\pm 1, 0), (-2, 0)$

Pg 5

Solve for x

trig $[0, 2\pi)$

(37) $x^4 + x^2 - 2 = 0$

$(x^2+2)(x^2-1) = 0$

$\boxed{x = \pm 1}$

Not Real

(38) $x^{1/2} + \frac{1}{x^{1/2}} - 2 = 0$

$\sqrt{x}\left(\sqrt{x} + \frac{1}{\sqrt{x}} - 2 = 0\right)$

$x + 1 - 2\sqrt{x} = 0$

$x - 2\sqrt{x} + 1 = 0$

$(\sqrt{x} - 1)(\sqrt{x} - 1) = 0$

$(\sqrt{x} - 1)^2 = 0$

$\sqrt{x} - 1 = 0$

$\sqrt{x} = 1$

$\boxed{x = 1}$

(39) $\left(15x - \frac{4}{x} = 4\right)x$

$15x^2 - 4 = 4x$

$15x^2 - 4x - 4 = 0$

$(5x+2)(3x-2) = 0$

$\boxed{x = -\frac{2}{5}, \frac{2}{3}}$ Check!

$15\left(-\frac{2}{5}\right) - \frac{4}{\frac{2}{5}} \stackrel{?}{=} 4$

$-6 + 10 \stackrel{?}{=} 4$

$4 = 4$ ✓

$15\left(\frac{2}{3}\right) - \frac{4}{\frac{2}{3}} \stackrel{?}{=} 4$

$10 - 6 = 4$

$4 = 4$ ✓

(40) $-x^5 + 29x^3 - 100x = 0$

$-x(x^4 - 29x^2 + 100) = 0$

$-x(x^2 - 4)(x^2 - 25) = 0$

$\boxed{x = 0, \pm 2, \pm 5}$

(41) $x^{2/3} + 2x^{1/3} - 15 = 0$

$(x^{1/3} + 5)(x^{1/3} - 3) = 0$

$x^{1/3} = -5 \qquad x^{1/3} = 3$

$x = (-5)^3 \qquad x = 3^3$

$\boxed{x = -125, 27}$

(42) $4x^2 - x^3 = 0$

$x^2(4-x) = 0$

$\boxed{x = 0, 4}$

(43) $-x^3 - 5x^2 + 4x + 20 = 0$

$-x^2(x+5) + 4(x+5) = 0$

$(x+5)(-x^2+4) = 0$

$(x+5)(4-x^2) = 0$

$\boxed{x = -5, \pm 2}$

(44) $-x^4 + x^3 + 20x^2 = 0$

$-x^2(x^2 - x - 20) = 0$

$-x^2(x-5)(x+4) = 0$

$\boxed{x = 0, 5, -4}$

(45) $\ln(x^2 - 3x - 5) = 0$

$e^0 = x^2 - 3x - 5$

$1 = x^2 - 3x - 5$

$x^2 - 3x - 6 = 0$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-6)}}{2(1)}$

$= \dfrac{3 \pm \sqrt{9+24}}{2}$

$\boxed{x = \dfrac{3 \pm \sqrt{33}}{2}}$

(46) $3 - \dfrac{6}{x-2} = 0 \quad ; x \neq 2$

$3 = \dfrac{6}{x-2}$

$3(x-2) = 6$

$x - 2 = 2$

$\boxed{x = 4}$

(47) $x^{5/2} - 4x^{3/2} - 5x^{1/2} = 0$

$x^{1/2}(x^2 - 4x - 5) = 0$

$x^{1/2}(x-5)(x+1) = 0$

$\boxed{x = 0, 5, \cancel{-1}}$ Not Real in equation

(48) $x^{7/3} + x^{4/3} - 12x^{1/3} = 0$

$x^{1/3}(x^2 + x - 12) = 0$

$x^{1/3}(x+4)(x-3) = 0$

$\boxed{x = 0, -4, 3}$

(49) $\ln(x^2+5x+7)=0$

$e^0 = x^2+5x+7$

$1 = x^2+5x+7$

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$\boxed{x=-3,-2}$

(50) $x^2e^x-3xe^x-10e^x=0$

$e^x(x^2-3x-10)=0$

$e^x(x-5)(x+2)=0$

$e^x \neq 0$ $\boxed{x=5,-2}$

(51) $\ln(5x)-\ln(x+2)=0 \quad ; x>0$

$\ln\frac{5x}{x+2}=0$

$e^0 = \frac{5x}{x+2}$

$1 = \frac{5x}{x+2}$

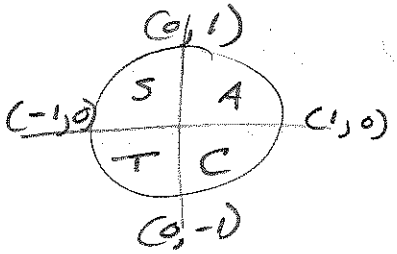
$x+2=5x$

$2=4x$

$\boxed{x=\frac{1}{2}}$

SOH-CAH-TOA

$[0, 2\pi)$



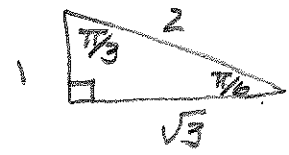
(52) $2\sin^2 x - \sin x = 0$

$\sin x (2\sin x - 1) = 0$

$\sin x = 0$ $\quad 2\sin x - 1 = 0$

$x = 0, \pi$ $\quad \sin x = \frac{1}{2}$

1st, 2nd quadrants (Sine is positive)



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$$

(53) $2\cos^2 x + \cos x - 1 = 0$

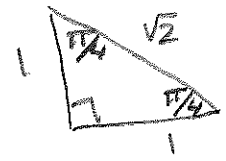
$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

1st, 4th quad. (cos is +) $\quad x = \pi$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$



(54) $\sin x + \cos x = 0$

$\sin x = -\cos x$

Same value, but opposite signs:

$\frac{\pi}{4}$, 2nd + 4th quads.

$$x = \frac{3}{4}\pi, \frac{7}{4}\pi$$

Common AP Question!

(55) $6\tan(2x) = 6$

$0 \le x < 2\pi$

$\frac{\pi}{4}$; 1st, 3rd quads $\quad \tan 2x = 1$

So, $0 \le 2x < 4\pi$ (twice around the unit circle)

$2x = \frac{\pi}{4}, \frac{5\pi}{4},$

Add 2π for second trip around circle $\quad \frac{9\pi}{4}, \frac{13\pi}{4} \Rightarrow$

Since $2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

(divide by 2)

then $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

$[0, 2\pi)$

(56) $2\sin x\cos x = \sqrt{3}\cos x$

$2\sin x\cos x - \sqrt{3}\cos x = 0$

$\cos x(2\sin x - \sqrt{3}) = 0$

$\cos x = 0$ $\quad \sin x = \frac{\sqrt{3}}{2}$ (1st, 2nd Quads.)

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad x = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

(57) $\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x + 1)(\sec x - 2) = 0$

$\sec x = -1$ $\quad \sec x = 2$

$\frac{1}{\cos x} = \frac{-1}{1}$ $\quad \frac{1}{\cos x} = \frac{2}{1}$

$\cos x = -1$ $\quad \cos x = \frac{1}{2}$

$x = \pi$ $\quad x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

(58) $2\sin^2 x - 3\cos x = 0$

$2(1 - \cos^2 x) - 3\cos x = 0$

Pythagorean Identity:

$\sin^2 x + \cos^2 x = 1$

$2 - 2\cos^2 x - 3\cos x = 0$

$-2 + 2\cos^2 x + 3\cos x = 0$

$2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x \neq -2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

Cosine is between -1 and 1, inclusive ALWAYS (as is sine)

(59) $4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\sqrt{\cos^2 x} = \pm\sqrt{\frac{3}{4}}$

$\cos x = \frac{\pm\sqrt{3}}{2}$ (all 4 quadrants)

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(60) $\tan^2\left(\frac{x}{2}\right) - 3 = 0$

$\tan\frac{x}{2} = \pm\sqrt{3}$

(Similar to #59)

$0 \le x < 2\pi$

So, $0 \le \frac{x}{2} < \pi$

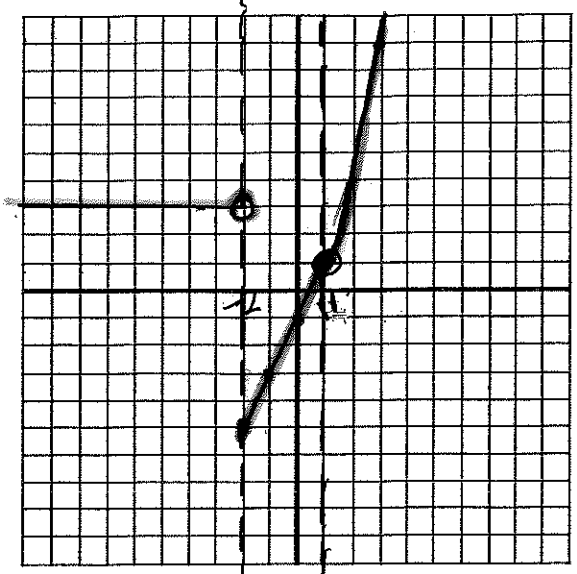
(only quads 1+2)

$\frac{x}{2} = \frac{\pi}{3}, \frac{2\pi}{3}$

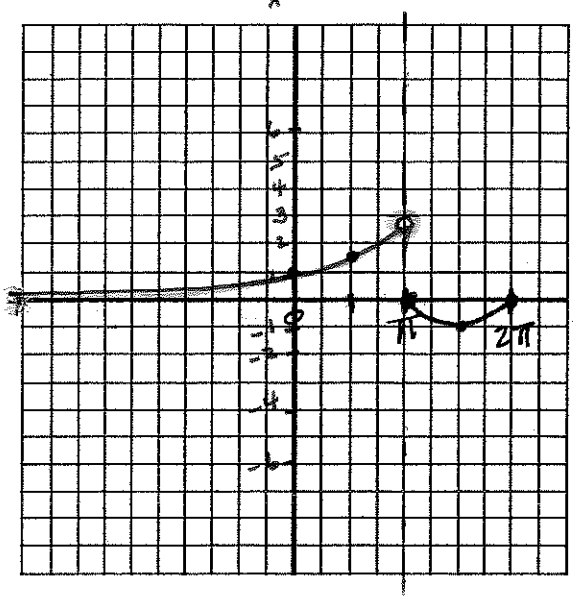
Multiply by 2 to eliminate the denominator of $\frac{x}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

(6) $f(x)=3,\ x<-2$ $\quad f(x)=-2x-1,\ -2\le x\le 1$ $\quad f(x)=x^2,\ x>1$



(62) $f(x)=e^{\frac{1}{2}x}$, $x<\pi$ $\quad f(x)=2\sin x$, $\pi\le x\le 2\pi$

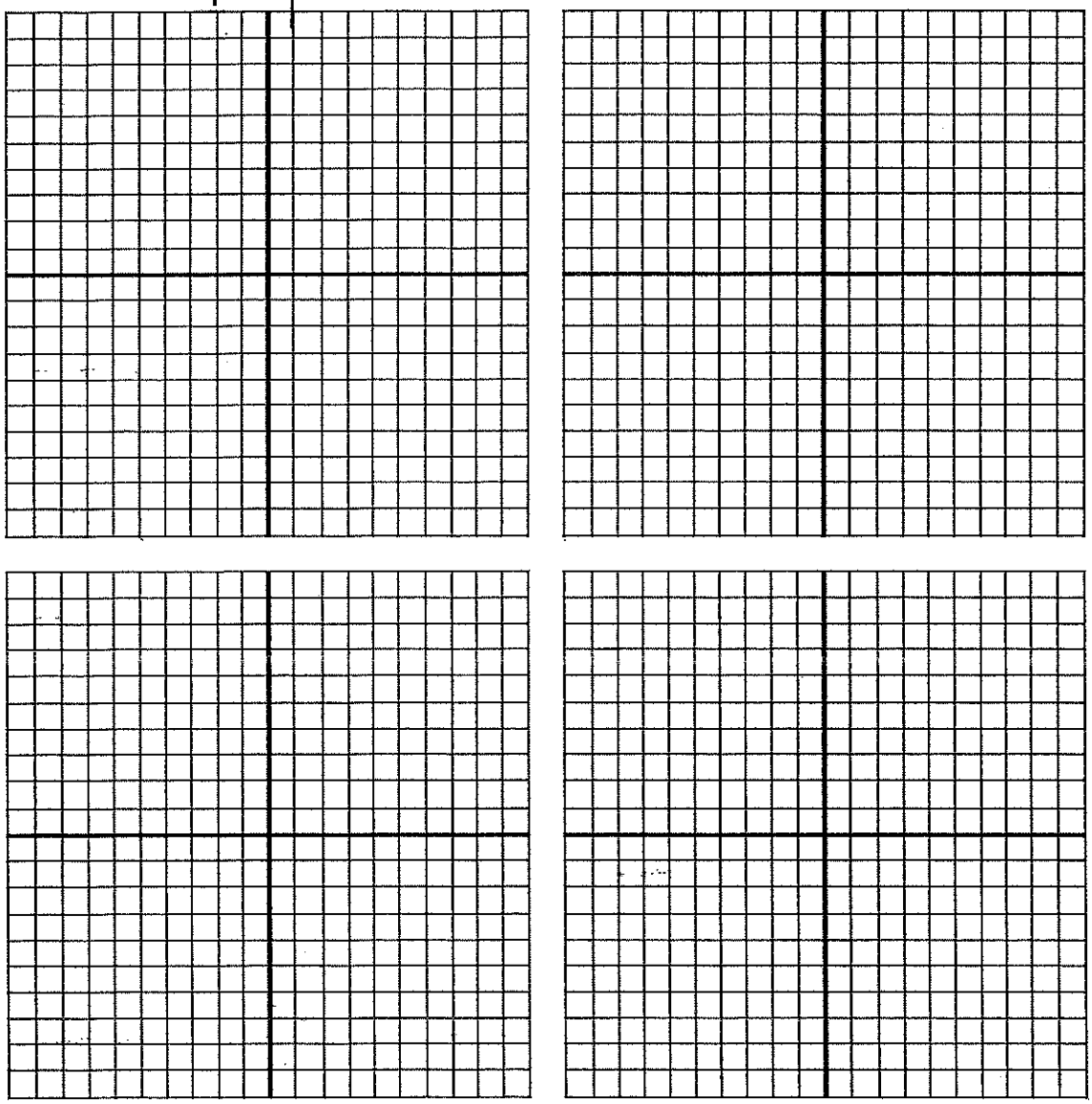


* $e^1=2.718$

$e^{\frac{1}{2}}=1.649$

$e^{-1/\pi}\simeq 0.727$

$e^{-100/\pi}\simeq 1.5\times10^{-14}$



EmbeddedMath.com

(63) f is increasing when the slope of the graph is positive:

on the open interval

$(-2.5, 1)$

f is decreasing when the slope of the graph is negative:

on the open intervals

$(-\infty, -2.5)$, $(1, \infty)$

* The points where $x = -2.5$ and 1 (the bottom and top of curves) are where the slope — if we draw tangent lines at those points — is zero.

The value of f refers to the y-coordinate at any point on the graph. The points where the value of f goes from negative (below x-axis) to positive (above x-axis) and vice versa occur at x-intercepts.

↑ This is just a fancy way to say "find the x-intercepts"!

$(x, 0)$: $(-4, 0)$, $(-1, 0)$, $(2, 0)$

(64) triangle $A = \frac{1}{2}bh$

$= \frac{1}{2}(4)(5)$

$= 10$ sq. units

trapezoid $A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(2)(16 + 7)$

$= 23$ sq. units

Area = $(10 + 23)$ sq. units

$= 33$ sq. units

Done!