① $f(x) = (x^6 + 2x)^{1/2}$

$f'(x) = \frac{1}{2}(x^6+2x)^{-1/2}(6x^5+2)$

$= \frac{1}{2}(x^6+2x)^{-1/2}\, 2(3x^5+1)$

$$\boxed{= \frac{3x^5+1}{\sqrt{x^6+2x}}}$$

② $g(x) = \cos(3x^{1/2})$

$g'(x) = \left[-\sin 3x^{1/2}\right]\left[3\left(\frac{1}{2}\right)x^{-1/2}\right]$

$$\boxed{= \frac{-3\sin 3\sqrt{x}}{2\sqrt{x}}}$$

③ $y = 4(\sin x)^3$

$y' = 4(3)(\sin x)^2[\cos x]$

$$\boxed{y' = 12\sin^2 x\cos x}$$

④ $f(x) = 4x(x^4 - x^2)^{\frac{1}{2}}$

$f = 4x \qquad g = (x^4 - x^2)^{\frac{1}{2}}$

$f' = 4 \qquad g' = \frac{1}{2}(x^4 - x^2)^{-\frac{1}{2}}(4x^3 - 2x)$

$+$

$$f'(x) = 4(x^4 - x^2)^{\frac{1}{2}} + 4x\left(\tfrac{1}{2}\right)(x^4 - x^2)^{-\frac{1}{2}}(4x^3 - 2x)$$

$$= 4(x^4 - x^2)^{\frac{1}{2}} + 2x(x^4 - x^2)^{-\frac{1}{2}}\, 2x(2x^2 - 1)$$

$$= 4(x^4 - x^2)^{\frac{1}{2}} + 4x^2(x^4 - x^2)^{-\frac{1}{2}}(2x^2 - 1)$$

$$= 4(x^4 - x^2)^{-\frac{1}{2}}\left[(x^4 - x^2)^1 + x^2(2x^2 - 1)\right]$$

$$= 4(x^4 - x^2)^{-\frac{1}{2}}\left[x^4 - x^2 + 2x^4 - x^2\right]$$

$$= 4(x^4 - x^2)^{-\frac{1}{2}}\left[3x^4 - 2x^2\right]$$

$$f'(x) = \frac{4x^2[3x^2 - 2]}{\sqrt{x^4 - x^2}}$$

(5) $y = (3x-4)^2(x^3+2)^3$

$f = (3x-4)^2 \qquad g = (x^3+2)^3$

$f' = 2(3x-4)^1(3) \qquad g' = 3(x^3+2)^2(3x^2)$

$= 6(3x-4) \qquad + \qquad = 9x^2(x^3+2)^2$

$$\frac{dy}{dx} = 6(3x-4)(x^3+2)^3 + 9x^2(3x-4)^2(x^3+2)^2$$

$$\frac{dy}{dx} = 3(3x-4)^1(x^3+2)^2\left[2(x^3+2)^1 + 3x^2(3x-4)^1\right]$$

$$= 3(3x-4)(x^3+2)^2\left[2x^3+4+9x^3-12x^2\right]$$

$$= 3(3x-4)(x^3+2)^2(11x^3-12x^2+4)$$

(6) $g(x) = 8x(\sin 3x^2)^3$

$f = 8x \qquad g = (\sin 3x^2)^3$

$f' = 8 \qquad g' = 3(\sin 3x^2)^2(\cos 3x^2)\,6x$

$+$

$$g'(x) = 8(\sin 3x^2)^3 + 8x(18x)(\sin 3x^2)^2(\cos 3x^2)$$

$$g'(x) = 8(\sin 3x^2)^2\left[\sin 3x^2 + 18x^2\cos 3x^2\right]$$

$$g'(x) = 8\sin^2(3x^2)\left[\sin(3x^2) + 18x^2\cos(3x^2)\right]$$

⑦ $y = \frac{(2x+1)^3}{(x-6)^3} = (2x+1)^3(x-6)^{-3}$

$f = (2x+1)^3 \qquad g = (x-6)^{-3}$

$f' = 3(2x+1)^2(2) \qquad g' = -3(x-6)^{-4}(1)$

$= 6(2x+1)^2 \qquad + \qquad = -3(x-6)^{-4}$

$\frac{dy}{dx} = 6(2x+1)^2(x-6)^{-3} + (-3)(2x+1)^3(x-6)^{-4}$

$\frac{dy}{dx} = 3(2x+1)^2(x-6)^{-4}\left[2(x-6)^1 - (2x+1)^1\right]$

$= 3(2x+1)^2(x-6)^{-4}\left[2x-12-2x-1\right]$

$= 3(2x+1)^2(x-6)^{-4}(-13)$

$= \frac{-39(2x+1)^2}{(x-6)^4}$ $\blacksquare$

OR

$$y = \frac{(2x+1)^3}{(x-6)^3}$$

$f = (2x+1)^3$ $\qquad$ $g = (x-6)^3$

$f' = 3(2x+1)^2(2)$ $\qquad$ $g' = 3(x-6)^2(1)$

$= 6(2x+1)^2$ $\qquad$ $= 3(x-6)^2$

$$\frac{dy}{dx} = \frac{6(2x+1)^2(x-6)^3 - 3(2x+1)^3(x-6)^2}{[(x-6)^3]^2}$$

$$= \frac{3(2x+1)^2(x-6)^2\left[\overset{2x-12}{2(x-6)} - \overset{-2x-1}{(2x+1)}\right]}{(x-6)^6}$$

$$= \frac{3(2x+1)^2(x-6)^2(-13)}{(x-6)^6}$$

$$= \frac{-39(2x+1)^2}{(x-6)^4}$$

⑧ $y = \cos(\cot 4x)$

$u = \cot 4x$ $\qquad$ $y = \cos u$

$\frac{du}{dx} = (-\csc^2 4x)4$ $\qquad$ $\frac{dy}{du} = -\sin u$

$= -4\csc^2 4x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= (-\sin u) \cdot (-4\csc^2 4x)$

$= (-\sin(\cot 4x))(-4\csc^2 4x)$

$= 4[\sin(\cot 4x)][\csc^2 4x]$

$= 4\csc^2(4x) \cdot \sin(\cot 4x)$

$\blacksquare$

⑨ $f(x) = 2x^3 \sin 4x^2$

$f = 2x^3$ $\qquad$ $g = \sin 4x^2$

$f' = 6x^2$ $\qquad$ $g' = 8x\cos 4x^2$

$+$

$f'(x) = 6x^2 \sin 4x^2 + 16x^4 \cos 4x^2$

$= 2x^2 [3\sin 4x^4 + 8x^2 \cos 4x^2]$

$\blacksquare$

(10) $y = 8x \sec^2(4x^2)$

$f = 8x \qquad g = \left(\sec 4x^2\right)^2$

$f' = 8 \qquad g' = 2\left(\sec 4x^2\right)^1 \left(\sec 4x^2 \tan 4x^2\right)(8x)$

$= 16x\left(\sec^2 4x^2\right)\left(\tan 4x^2\right)$

$$\frac{dy}{dx} = 8\sec^2 4x^2 + 8x(16x)\left(\sec^2 4x^2\right)\left(\tan 4x^2\right)$$

$$= 8\sec^2 4x^2\left[1 + 16x^2 \tan 4x^2\right]$$